東吳大學 103 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (20 分) In each case, find a function satisfying the given condition.

   (a) $f'(x^2) = x$ for $x > 0$, $f(1) = 1$.

   (b) $f'(\ln x) = \begin{cases} 1 & 0 < x \le 1, \\ x & x > 1, \end{cases}$ $f(0)=0$

2. (20 分) Let $f(x, y) = \dfrac{xy^2}{x^2 + y^4}, x \neq 0$, $f(0, y) = 0$.

   (a)Compute the directional derivative of f at (0,0) in a given direction $\vec{u} = (u_1, u_2)$, $\vec{u}$ is a unit vector.

   (b) Is f differentiable at (0, 0)? (You should give reason for your answer)

3. (20 分) Let $f(x) = \dfrac{1}{1-x}, x \neq 1$. Let $S(x)$ be the Taylor series of $f(x)$ centered at $x = 0$ and $S_n(x)$ be the nth partial sum of S(x).

   (a) S(x) =? $E(x) = f(x) - S_n(x) = ?$

   (b) Describe the region such that $S_n(x)$ converges uniformly to $S(x)$. (you should give reason for your answer)

4. (20 分) Find the points (x, y) and the directions for which the directional derivative of $f(x, y) = 3x^2 + y^2$ has its largest value, if $f(x, y)$ is restricted to be on the circle $x^2 + y^2 = 1$.

5. (20 分) Let $\Gamma(s) = \int_{0^+}^{\infty} t^{s-1} e^{-t} dt$.

   (a) Prove that the domain of $\Gamma(s)$ is $s > 0$.

   (b) Prove that $\Gamma(n+1) = n!$ for every positive number n.