東吳大學 103 學年度碩士班研究生招生考試試題

系級	數學系碩士班 B 組	考試時間	100 分鐘
科目	基礎數學	本科總分	100 分

1. (10%) Find $\lim_{x\to\infty}(\cos\frac{1}{x})^x$.

2. (10%) Find the equation for the tangent line to the curve

$$2\sin(x+y) = 2xy + 1$$

at the point $(0, \frac{\pi}{6})$.

3. (20%) Evaluate the following integrals.

$$\text{(a) } \int \frac{1}{x^3+1}dx. \quad \text{(b) } \int e^{2x}\sin x dx.$$

4. (10%) Determine whether the series $\sum_{k=2}^{\infty}\frac{1}{k\ln k}$ converges or diverges.

5. (20%) Let the transformation T be a reflection across the line $y = x$ in the plane.

(a) Find the matrix M_{B_1} of T with respect to the basis $B_1 = \{(1,0),(0,1)\}$.

(b) Find the matrix M_{B_2} of T with respect to the basis $B_2 = \{(1,1),(1,-1)\}$.

(c) Show that M_{B_1} and M_{B_2} are similar.

6. (30%) Let $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Show that A has linearly independent columns.

(b) Apply Gram-Schmidt process to find an orthogonal basis for the column space of A.

(c) Find the QR-factorization of A.