

東吳大學 103 學年度碩士班研究生招生考試試題

第 1 頁，共 3 頁

系級	經濟學系碩士班	考試時間	100 分鐘
科目	統計學	本科總分	100 分

1. (12 points) Please find the means and variances of the following probability distributions:

(1) $f(x) = \binom{35}{x} \binom{65}{25-x} / \binom{100}{25}, x = 0, 1, 2, \dots, 25$

(2) $f(x) = 6e^{-6x}, x \geq 0$

(3) $f(x) = a^x e^{-a} / x!, a > 0, x = 0, 1, 2, \dots$

2. (12 points) Please derive the maximum likelihood estimate of θ for each of the following cases:

(1) $f(x|\theta) = \theta x^{\theta-1}, 0 < x < 1, 0 < \theta < \infty$, zero elsewhere.

(2) $f(x|\theta) = \frac{\theta^x}{x!} e^{-\theta}, \theta > 0, x = 0, 1, 2, \dots$

3. (12 points) Your answers in the following questions should be made in *inequalities*.

(1) A random variable X is distributed with mean 60 and variance 144. Please find the probability for the interval of x values, $36 \leq x \leq 96$, assuming the probability distribution is symmetric around mean.

(2) Suppose that the z-scores of the distribution in part (1) follow a Student's t distribution with some degrees of freedom. Please find the probability for the interval.

(3) Is your answer in part (2) consistent with that in part (1)?

4. (12 points) Consider two random variables, X and Y. You obtained a sample of 20 pairs of observations randomly selected from their joint probability distribution. There are 4 observations for $y = 2$, 10 observations for $y = 4$, and 6 observations for $y = 8$. The condition probabilities for X given Y are shown in the following table:

X \ Y	2	4	8
2	1/2	2/5	0
4	1/2	2/5	1/3
8	0	1/5	2/3

(1) Please calculate probabilities $P(X)$, $X = 2, 4$, and 8.

(2) Please calculate sample covariance.

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5. (16 points) The Earthquake Hazards Program of the United States Geological Survey provides the following statistics data on worldwide large earthquakes with magnitude 7 and greater over a certain period of 16 years:

Occurrences		Deaths	
Total	Monthly Average	Total	Annual Average
220	1.15	545,101	34,069

- (1) We are concerned with the probability of various occurrences (denoted by X) in the coming month. What will be an appropriate probability distribution to describe it? Please write down its mathematical expression clearly.
- (2) Based on the probability distribution in part (1), calculate the probability $P(X > 1)$ in the coming year.
- (3) We are concerned with the time interval (denoted by Y) between earthquakes. What will be an appropriate probability distribution to describe it? Please write down its mathematical expression clearly.
- (4) Based on the probability distribution in part (3), calculate the probability of occurring an earthquake over the future period of 1 to 2 months.

6. (24 points) A production function model is specified as

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \mu_i$$

where $Y_i = \log$ output, $X_{2i} = \log$ labor input, and $X_{3i} = \log$ capital input. The data refer to a sample of 23 firms, and observations are measured as deviations from the sample means

$$\begin{aligned} \sum X_{2i}^2 &= 12 & \sum X_{2i}X_{3i} &= 8 \\ \sum X_{3i}^2 &= 12 & \sum Y_i X_{2i} &= 10 \\ \sum Y_i^2 &= 10 & \sum Y_i X_{3i} &= 8 \end{aligned}$$

- (1) (10 points) Estimate β_2 , β_3 , their standard errors, and R^2 .
- (2) (6 points) Test the hypothesis that $\beta_2 + \beta_3 = 1$.
- (3) (8 points) Suppose now that you wish to impose the a priori restriction that $\beta_2 + \beta_3 = 1$. What is the least-squares estimate of β_2 and its standard error?

7. (12 points) 以下的 Excel 迴歸摘要輸出是利用美國 1980/01 至 2006/07 的月資料，分析消費函數的

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結果：

摘要輸出

迴歸統計	
R 的倍數	0.99880615
R 平方	0.99761372
調整的 R 平方	0.99759862
標準誤	103.97235
觀察值個數	319

ANOVA

	自由度	SS	MS	F	顯著值
迴歸	2	1428119586	714059793	66053.96	0
殘差	316	3416038.87	10810.2496		
總和	318	1431535625			

	係數	標準誤	t 統計	P-值	下限 95%	上限 95%
截距	-404.09353	36.5531714	-11.0549514	0	-476.0119	-332.1752
X 變數 1	0.86956469	0.00357383	243.314625	0	0.862533	0.8765962
X 變數 2	5.66321126	2.99612428	1.89017902	0.059649	-0.231662	11.558084

表中“X 變數 1”指所得，“X 變數 2”指利率。

- (1) 根據上表，F 檢定的結果是指甚麼？請先列出檢定假設 H_0 與 H_a 。
- (2) 根據上表，t 檢定是否顯著？與第(1)小題的結果是否一致？請說明。