

東吳大學 103 學年度轉學生(含進修學士班轉學生)招生考試試題

第 1 頁, 共 1 頁

系級	財務工程與精算數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (10%) Evaluate the limit.

$$\lim_{x \rightarrow +\infty} \left(1 + \sin \frac{3}{x}\right)^x$$

2. (10%) Find all c that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \cos x - \sin x$ on the interval $[0, 2\pi]$.

3. (10%) Evaluate the indefinite integral.

$$\int \frac{x}{4x^2 + 12x + 13} dx$$

4. (10%) Evaluate the following integral or show that it diverges.

$$\int_0^1 \ln x dx$$

5. (10%) Show that the function $u(x, y) = e^{2x} \sin(2y)$ satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

6. (10%) Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3 + 4n + 3}}$$

7. (10%) Evaluate the limit.

$$\lim_{n \rightarrow +\infty} \left(\frac{1}{\sqrt{n}\sqrt{n+1}} + \frac{1}{\sqrt{n}\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n}\sqrt{n+n}} \right)$$

8. (10%) Use Lagrange multiplier to find the volume of the largest rectangular box in the first octant (卦限) with three faces in the coordinate planes and one vertex (頂點) in the plane $x + 2y + 3z = 6$.

9. (10%) Prove that $\log_2 5$ is an irrational number.

10. (10%) Use the transformation $u = x - y$, $v = x + y$ to evaluate the integral

$$\iint_R \frac{x-y}{x+y} dA,$$

where R is the square with vertices $(0, 2)$, $(1, 1)$, $(2, 2)$, and $(1, 3)$.