

東吳大學 104 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (10 points) Determine whether the set S is linearly dependent or independent.

$$S = \{(1, 0, 1, 0), (0, 3, 0, 1), (1, 1, 2, 2), (3, -4, 2, -3)\}$$

2. (16 points) Find bases for the four fundamental subspaces of the matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

3. (16 points) Apply the Gram-Schmidt process to transform the given basis

$$B = \{(1, 1, 2), (1, 1, 1), (1, 0, 1)\}$$

for \mathbb{R}^3 into an orthonormal basis.

4. (16 points) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T(1, 1, 1) = (2, 0, -1), \quad T(0, -1, 2) = (-3, 2, -1), \quad T(1, 0, 1) = (1, 1, 0).$$

Find $T(2, -1, 1)$.

5. (16 points) Determine whether the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 + x_3)$$

is invertible. If it is, then find its inverse.

6. (16 points) Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. Find a nonsingular matrix P and a diagonal matrix Λ such that $P^{-1}AP = \Lambda$.

7. (10 points) Let A be a square matrix. Prove that if $A^2 = A$ then $I - 2A$ is invertible and $(I - 2A)^{-1} = I - 2A$.