

東吳大學 104 學年度轉學生(含進修學士班轉學生)招生考試試題

第 1 頁, 共 1 頁

系級	數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (20%) (a) On the curve $xe^y + \sin(xy) + y - \ln 2 = 0$, viewing y as a differentiable function of x , find $\frac{dy}{dx}$ and the tangent line at $(0, \ln 2)$.
- (b) Suppose $(0.1, \tilde{y})$ is on this curve, estimate \tilde{y} .
2. (20%) (a) Suppose $f'(x)$ exists and is continuous on $[a, b]$, explain that the arc length of the graph of $y = f(x)$ over $[a, b]$ is equal to $\int_a^b \sqrt{1 + [f'(x)]^2} dx$.
- (b) Calculate the arc length of the graph of $f(x) = \ln(\cos x)$ over $[0, \frac{\pi}{4}]$.
3. (20%) Find the gradient of $f(x, y) = \tan^{-1} \frac{\sqrt{x}}{y}$ at $(4, -2)$. Also sketch the gradient vector together with the level curve that passes through $(4, -2)$.
3. (20%) Sketch $D = \{(x, y) \mid x^2 + y^2 \leq 1, y \geq \frac{1}{2}\}$ and integrate $f(x, y) = \frac{y}{x^2 + y^2}$ over D using polar coordinates.
4. (20%) What is the radius of convergence of $\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{x^n}{n}$? What is the domain of $f(x) = \sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{x^n}{n}$? Also show that $f(x) \equiv \ln(1+x)$ in its domain.