

東吳大學 104 學年度轉學生(含進修學士班轉學生)招生考試試題

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系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (15%) Let $f : S \subset \mathbb{R}^n \rightarrow \mathbb{R}$, where S is open and $\vec{a} \in S$. Prove that if f has a local maximum or minimum at \vec{a} and f is differentiable at \vec{a} , then $\nabla f(\vec{a}) = \vec{0}$

2. (15%) $\vec{f}(x, y, z) = (2x + (y-1)^2 - \sin z, 3x + 2e^{2y-5z})$.

a. Compute $D\vec{f}(x, y, z)$. What are $\vec{f}(0,0,0)$ and $D\vec{f}(0,0,0)$?

b. Suppose $\vec{g} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is of class C^1 and $D\vec{g}(1,2) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

Compute $D(\vec{g} \circ \vec{f})(0,0,0)$.

3. (15%) Let $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$. Show that f is continuous at $x=0$ and nowhere else.

4. (15%) Let $f(x) = 1$ if x is rational, $f(x) = 0$ if x is irrational.

(a) What is the set of points in $[0,1]$ at which f is discontinuous?

(b) Show that f is not integrable on $[0,1]$.

5.(a) (10%) Evaluate $\iint_S (x - \sqrt{y}) dA$, S : the region between the parabola $x = y^2$ and the line $x = y$.

(b)(10%) Evaluate $\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3 + 1) dy dx$

6.(a)(10%) Let $f_n(x) = x^{1/n}$. For $x > 0$, find $\lim_{n \rightarrow \infty} x^{1/n}$ and find f such that f_n converges pointwise to f on $[0, \infty)$.

(b) (10%) Let $f(x) = \sum_{k=1}^{\infty} \frac{\cos(kx)}{k^2}$. Show that f is convergent uniformly on \mathbb{R} .