東吳大學 104 學年度轉學生(含進修學士班轉學生)招生考試試題

系級	數學系三年級	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (25%) Let

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 3 & 1 & -5 & 0 \\ 1 & 2 & 0 & -5 \end{bmatrix}$$

a. Use Gaussian elimination to find the complete solution of

$$A\mathbf{x} = (5, 8, -9)^T.$$

b. Give a basis for the column space of A.

c. Give a basis for the nullspace of A.

2. (25%) Find invertible matrix S and diagonal matrix Λ so that

$$A = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} = S\Lambda S^{-1}.$$

Then find $\lim_{k\to\infty} A^k$.

3. (25%) Let $T : I\!R^3 \to I\!R^3$ be a linear transformation satisfying

$$T(1,1,0) = (3,-1,3), \quad T(0,1,1) = (2,1,3), \quad T(0,0,1) = (-1,-2,0).$$

Find the standard matrix representation for T and $T(x, y, z)$.

4. (25%)

a. Apply the Gram-Schmidt process to transform the basis

$$\mathbf{B} = \{(1,1,0), (1,2,0), (0,1,2)\}$$

for $I\!R^3$ into an orthonormal basis.

b. Find QR-factorization of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.