東吳大學 105 學年度轉學生(含進修學士班轉學生)招生考試試題

系級	數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1.(12%) Find the limit.

(a) $\lim_{x\to 1}\frac{3x-3}{\sqrt{x+3}-2}$ (b) $\lim_{x\to 0^+} x\ln x$ (c) $\lim_{x\to 0^+}(\frac{1}{x}-\frac{1}{e^x-1})$

2. (12%) Sketch the graph of $y=x^3-6x^2+9x+2$.

3. (20%) [1] Find the derivative.(a) $y=\int_0^x \sqrt{t^3+1}dt$ (b) $y=10^x+\tan x^3$.

[2] If $u=f(x,y)$, $x=u-v$, $y=v-u$, show that

$$\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}=0.$$

4. (20%) Evaluate (a) $\int_0^{\pi/2}\sin^5 x\cos x dx$ (b) $\int_1^5 f''(x)dx$

(c) $\int_0^1 \frac{2x+1}{x^2+1}dx$ (d) $\int x\cos x dx$

5. (10%) Find the area of the region bounded by $x=y^2$ and $y=x-2$.

6.(12%) Let $z=f(x,y)=x\sin(3x-2y)$.

(a) Find the gradient $\nabla f(x,y)$

(b) Find the equation of the tangent plane to the graph of f at the point where $x=1$ and $y=1$.

7. (14 %) Let $f(x,y)=x^2+5y^2+x^2y+2y^3$

(a) Find the critical points of f.

(b) Classify the relative extrema and saddle points of f.