

東吳大學 105 學年度轉學生(含進修學士班轉學生)招生考試試題

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系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (20 points) Let $f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$ Evaluate $f'(x)$, and show that $f'(x)$ is discontinuous at $x = 0$.
2. (20 points) Give a reason that there does not exist a continuous functions f such that $f([-1, 1]) = (-1, 1)$, and no continuous function g such that $g([0, 3]) = [0, 1] \cup [2, 4]$.
3. (20 points) Let $f : [a, b] \rightarrow \mathbf{R}$ be a continuous function, and $\int_a^b f^2(t) dt = 0$. Prove that $f(x) = 0$ on $[a, b]$.
4. (20 points) Prove $f_n(x) = x^n$ converges uniformly on $[0, 0.9]$, but not converges uniformly on $[0, 1]$.
5. (20 points) State the following theorems (no proofs):
 - (1) Intermediate value theorem.
 - (2) Mean value theorem.
 - (3) Bolzano-Weierstrass theorem.
 - (4) Heine-Borel theorem.
 - (5) Ratio test for a series.