東吳大學 105 學年度轉學生(含進修學士班轉學生)招生考試試題

系級	經濟學系三年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (10 points) Find the extreme value(s) of $z = 2x_1^2 + x_1x_2 + 4x_2^2 + x_1x_3 + x_3^2 + 2$.

2. (20 points) Use the Lagrange-multiplier method to find the stationary values of z and determine whether it is a maximum or a minimum:

(a) $z = xy$, subject to $x + 2y = 2$.
(b) $z = x(y + 4)$, subject to $x + y = 8$.

3. (10 points) Determine whether $q = 4u^2 + 4uv + 3v^2$ subject to $u - 2v = 0$ is either positive or negative definite.

4. (20 points) Find the Maclaurin series (with $n = 4$ and $x_0 = 0$) and Taylor series (with $n = 4$ and $x_0 = -2$) for:

(a) $\phi(x) = \dfrac{1}{1-x}$.

(b) $\phi(x) = \dfrac{1-x}{1+x}$.

5. (10 points) The following three equations

$xy - w = 0 \quad F^1 = (x, y, w; z) = 0$
$y - w^3 - 3z = 0 \quad F^2 = (x, y, w; z) = 0$
$w^3 + z^3 - 2zw = 0 \quad F^3 = (x, y, w; z) = 0$

are satisfied at point $P : (x, y, w; z) = (1/4, 4, 1, 1)$.

(a) Check whether the conditions of the implicit-function theorem are satisfied.
(b) Use the implicit-function theorem to find the comparative-static derivative ($\partial x/\partial z$).

背面尚有試題

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6. (10 points) Use Cramer's rule to solve the following equation systems:

(a)
$$\begin{aligned} 4x + 3y - 2z &= 1 \\ x + 2y &= 6 \\ 3x + z &= 4 \end{aligned}$$

(b)
$$\begin{aligned} -x + y + z &= a \\ x - y + z &= b \\ x + y - z &= c \end{aligned}$$

7. (10 points) Are the following functions strictly monotonic?

(a) $y = -x^6 + 5 \quad (x > 0)$

(b) $y = 4x^5 + x^3 + 3x$

Find dx/dy by the inverse-function rule.

8. (10 points) Given $A = \begin{bmatrix} -1 & 5 & 7 \\ 0 & -2 & 4 \end{bmatrix}$, $b = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$, and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Calculate (a) Ab, (b) AIb, (c) $x'IA$, and (d) $x'A$.