東吳大學 106 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. 20% Suppose $f(x)$ is a continuous function on $[a, b]$, $f(a) < 0 < f(b)$, and $f(x)$ has only one zero x_0 in (a, b). If we use bisection method to find x_0, that is, if $f(\frac{a+b}{2}) < 0$, define $a_1 = \frac{a+b}{2}$, $b_1 = b$; if $f(\frac{a+b}{2}) > 0$, define $a_1 = a$, $b_1 = \frac{a+b}{2}$, then repeat this process inductively to get a_n, b_n. Suppose a_n, b_n are well defined for each positive integer n.

(i) Prove that both $\{ a_n \}$ and $\{ b_n \}$ converge as $n \to \infty$.

(ii) Use $\lim_{n\to\infty} a_n$ and $\lim_{n\to\infty} b_n$ to represent x_0.

(iii) If we use a_n to approximate x_0, estimate the error.

2. 10% Suppose $f(x, y)$ is continuous in $\Re^2$, $f(0,0) > 0$, $f(1, 1) < 0$, will there exist a point (x_0, y_0) such that $f(x_0, y_0) = 0$? Why?

3. 10% $\int_0^\infty e^{-x^2} dx = ?$

4. 20% Suppose $f(x)$ is differentiable and $c \neq 0$ is a constant, let $u(x, t) = f(x - ct)$.

(i) Show that $u(x, t)$ satisfies $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$.

(ii) Show that ∇u (i.e. gradient of u) is orthogonal to $(c, 1)$ everywhere.

(iii) What are the level curves of $u(x, t)$? Moreover, use the result in (ii) to confirm your answer.

5. 20% (i) What is the domain of the function $f(x) = \sum_{n=0}^{\infty} x^n$. (You have to give some reason of your answer)

(ii) What is the domain of the function $f'(x)$? (You have to give some reason of your answer)

6. 20% Find the extreme values of $f(x, y) = x - x^2 y^3$ on Ω, where $\Omega = \{(x, y) \mid -1 \le x \le 1, -1 \le y \le 1\}$.