東吳大學106學年度碩士班研究生招生考試試題

系級	數學系碩士班A組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (20%)

(a) Let M = $\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}$, where a+b+c+d=0. Find det(M).

(b) Let N = $\begin{bmatrix} x & 0 & 0 & 0 \\ 1 & y & 0 & 0 \\ 0 & 1 & z & 0 \\ 0 & 0 & 1 & w \end{bmatrix}$, where $xyzw \neq 0$. Find det(N).

(c) Are the matrices M and N invertible? If it is invertible, find it's inverse matrix.

2. (20%)

(a) Let W_1 and W_2 are subspace of finite dimensional vector space V. Show that $W_1 \cap W_2$ is also subspace of V.

(b) Is $W_1 \cup W_2$ subspace of V? If true, prove it; if false, given a counterexample.

(c) Given the smallest subspace of V containing $W_1 \cup W_2$.

3 (20%)

(a) Show that the set {(0,1,1),(1,0,1),(1,1,0)} is basis for vector space R^3

(b) Let T : $R^3 \to R^4$ be linear transformation with T(0,1,1) = (2,1,0,1), T(1,0,1) = (2,1,1,0) and T(1,1,0) = (2,0,1,1), find T(x,y,z).

(c) Find the kernel of T, given the nullity of T and rank of T.

4 (20%) Apply the Gram-Schmidt process to transform the basis vectors u_1 = (0,1,1), u_2 = (1,0,1), u_3 = (1,1,0) into an orthogonal basis {v_1, v_2, v_3}.

5 (20%) Let A = $\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A.

(b) Find an invertible matrix C and a diagonal matrix D such that $D = C^{-1} A C$.

(c) Find r, s, t, w such that $A^{100} = \begin{bmatrix} r & s \\ t & w \end{bmatrix}$