

東吳大學 106 學年度轉學生(含進修學士班轉學生)招生考試試題

第 1 頁，共 1 頁

系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (10%) Show that if  $S_1$  and  $S_2$  are open subsets in  $R^n$ , so is  $S_1 \cap S_2$ .

2. (10%) Let  $g(x, y) = \frac{x^2 y}{x^4 + y^2}$  for  $(x, y) \neq (0, 0)$  and  $g(0, 0) = 0$

Show that  $\lim_{(x,y) \rightarrow (0,0)} g(x, y)$  does not exist.

3. (10%) Let  $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$  and  $x_0 \neq \pm 1$ . Show that  $f$  is discontinuous at  $x_0$ .

4. (10%) Prove that if  $f$  has a local maximum or minimum at  $\vec{a} \in R^n$  and  $f$  is differentiable at  $\vec{a}$ , then  $\nabla f(\vec{a}) = \vec{0}$

5. (15%) Find the extreme value of  $f(x, y) = 3x^2 - 2y^2 + 2y$  on the set  $\{(x, y) : x^2 + y^2 \leq 1\}$ .

6. (15%) Can the equations  $xy^2 + xzu + yv^2 = 3$ ,  $u^3 yz + 2xv - u^2 v^2 = 2$  for  $u$  and  $v$  as functions of  $x$ ,  $y$ , and  $z$  near  $x = y = z = u = v = 1$ .

7. (10%)(a) Evaluate  $\int_1^2 \int_0^1 3x^2 y dx dy$ .

(b) Let  $S = \{(x, y) : y \geq 0, x^2 + y^2 \leq 1\}$ . Evaluate  $\iint_S y dA$ .

8. (10%) Find the volume of the region in  $R^3$  above the triangle  $T$  with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(1, 2)$  and below the surface  $z = xy + y^2$ .

9. (10%) Determine whether the series  $\sum_{n=2}^{\infty} (-1)^n \log\left(\frac{n+1}{n}\right)$  converges absolutely, conditionally, or diverges.