東吳大學 106 學年度轉學生(含進修學士班轉學生)招生考試試題

系級	數學系三年級	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (a) Let A and B be 4×4 matrices with det A = 2 and det B = 3. Find det (AB^T), det (3A) and det $(A^{-1}B)$

   (b) Let $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \\ 0 & 6 & 0 & 0 \end{bmatrix}$. Find det A

   (d) Let A and B be two 3×3 matrices. if $(AB)^{-1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 7 & 3 \end{bmatrix}$, and $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find B.

   (e) Find the matrix A in the following equation: $A^T - \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^T \begin{bmatrix} 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$

   (f) Let x, y $\in R^n$. If ||x|| = 3, ||y|| = 1 and x·y = -2, then (x-2y)·(3x+5y) = ? (25%)

2. (a) To determine that the set {[1,-3,2], [2,-5,3], [4,0,1]} in R^3 is linear dependent or linear independent.

   (b) Let V be a vector space and $v_1, v_2, v_3 \in V$. Prove that if $\{v_1, v_2, v_3\}$ are linear independent then $\{v_1, v_1 + v_2, v_1 + v_2 + v_3\}$ are linear independent. (20%)

3. (a) Let T: V→ W be a linear transformation. Prove that T is 1-1 if and only if ker T = {0}.

   (b) Let T: $\mathbb{R}^\infty \to \mathbb{R}^\infty$, where $T(x_1, x_2, \cdots, x_n, \cdots) = (0, x_1, x_2, \cdots, x_n, \cdots)$. Is T 1-1? Is T onto? Prove it (15%)

4. Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$. (a) Find the eigenvalues and eigenvectors of A. (b) Find an invertible matrix C and a diagonal matrix D such that $D = C^{-1} A C$. (c) Find an expression for A^n. (20%)

5. Let T: $\mathbf{P_2} \to \mathbf{P_2}$ be linear transformation defined by $T(p(x)) = D^2(p(x)) - 4D(p(x)) + 2p(x)$. Let $\boldsymbol{\beta} = (1, x, x^2)$ be ordered basis of $\mathbf{P_2}$.

   (a) Find $[T]_\beta$ i.e. the matrix representation of T relative the ordered basis β.

   (b) If $[p(x)]_\beta = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, find T(p(x)) (20%)