

系級	財務工程與精算數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

要有演算過程或寫出理由, 否則扣分.

1. (10%) Evaluate the limit.

$$\lim_{x \rightarrow +\infty} (\sqrt{9x^2 + x} - 3x)$$

2. (15%) Find the radius (半徑) of convergence and interval (區間) of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$$

3. (15%) Evaluate the indefinite integral.

$$\int \sqrt{4x - x^2} dx$$

4. (15%) Find an equation of the tangent line to the curve

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

at the point $(-1, \frac{3\sqrt{3}}{2})$.

5. (15%) Suppose that f and g are continuous on the interval $[a, b]$ and differentiable on (a, b) . Furthermore, suppose that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Apply the mean value theorem to prove that $f(x) < g(x)$ for all $x \in (a, b]$.

6. (15%) Use polar coordinates to find the volume of the region (區域) above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

7. (15%) Use Lagrange multiplier to find the maximum and minimum values of the function

$$f(x, y, z) = xy^2z$$

subject to the constraint (限制式) $x^2 + y^2 + z^2 = 4$.