

系級	財務工程與精算數學系三年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

請詳述計算過程與理由，僅寫答案者不予計分

1. (10%) Find the limit

$$\lim_{x \rightarrow 27} \frac{\sqrt{1+x^{1/3}} - 2}{x - 27}.$$

2. (10%) Find the first derivative of the function f [setting $y = f(x)$] defined by the equation

$$\frac{y}{x+y} = 1 - x^2$$

at the point $(1,0)$.

3. (15%) Find the maximum points, the minimum points and points of inflection of the graph of the given function

$$G(x) = \frac{e^x - 1}{e^x + 1} \text{ on } [-1,1].$$

4. (10%) Show that if an initial amount A_0 of money is invested at an interest rate r compounded continuously a year, then the amount after t years is

$$A = A_0 e^{rt}.$$

5. (10%) Find the derivative of the function

$$F(x) = \int_x^0 \sqrt{1 + \sec t} dt.$$

6. (10%) Evaluate the integral

$$\int_{-\infty}^0 x e^{2x} dx.$$

7. (10%) Find the first three nonzero terms in the Maclaurin series for $e^{2x} \sin x$.

8. (10%) Evaluate the integral

$$\int \frac{1}{t^2 + 3t - 4} dt.$$

9. (15%) Evaluate $\iint_D 2x \sin y dA$, where D is the region bounded by the parabolas $y = 3x^2$ and $y = 2 + x^2$.