

東吳大學 107 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. 20% Use the following linear system to verify that the general solution of a consistent linear system $Ax=b$ can be obtained by adding any specific solution of $Ax=b$ to the general solution of $Ax=0$.

$$x + 2y - 3z = 3$$

$$3x + 6y - 9z = 9$$

$$-2x - 4y + 6z = -6$$

2. 20%

(a) Find the standard matrix for the transformation $T(x_1, x_2, x_3) = (x_1 - 2x_2, x_2 + 3x_3, 2x_1 - x_3)$ in \mathbf{R}^3 .

(b) Find the standard matrix for the reflection about the line $y=x$ in \mathbf{R}^2 .

(c) Suppose T is a linear transformation from \mathbf{R}^2 to \mathbf{R}^2 , and transforms $(1,2)$ to $(-3,4)$ and $(2,0)$ to $(1,5)$.

Find $T(v)$, $T^2(v)$ for $v = (3, -2)$.

3. 20%

(a) Let E be an $n \times n$ elementary matrix, what is the determinant $\det(E)$ of E ?

(b) Determine whether the set W of all 2×2 symmetric matrices is a vector subspace of M_{22} . Is it a finite-dimensional vector subspace?

4. 20% For $A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 6 & 2 & 0 \\ -1 & -3 & 0 & 4 \end{bmatrix}$, find the reduced row echelon form of A and verify the dimension

theorem for matrices .

5. 20% Let \mathbf{R}^3 have the Euclidean inner product, and let $v_1 = (1, 0, 0)$, $v_2 = (1, 0, -1)$, $v_3 = (1, 1, 0)$. Use the Gram-Schmidt process to find the QR-decomposition of $A = [v_1 \ v_2 \ v_3]$.