

系級	數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

CALCULUS

- (4 points for each) Compute the following limits:
 - $\lim_{x \rightarrow 5} (x^3 + x^2 + 1)$;
 - $\lim_{x \rightarrow 0} \frac{\sin(5x)}{3x}$;
 - $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$;
 - $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x + x^2}$;
 - $\lim_{x \rightarrow \infty} \frac{x^3}{e^x}$.
- (5 points for each) Find $\frac{dy}{dx}$:
 - $y = e^x$;
 - $y = \sin(x)$;
 - $y = \frac{x^2 - 1}{x^3 + 1}$;
 - $y = (2x + 1)^3 + 3(2x + 1)^2 + 1$;
 - $y^2 = x^2 + \sin(xy)$.
- (10 points) Find the absolute maximum and minimum values of $f(x) = 10x(2 - \ln(x))$ on the interval $[1, e^2]$.
- (6 points) Show that if $f(x)$ is a function such that $f'(x) = 0$, then $f(x)$ must be a constant function.
- (5 points for each) Compute the following indefinite integrals:
 - $\int \cos(2x) dx$;
 - $\int \sqrt{x} dx$;
 - $\int x\sqrt{x^2 - 1} dx$;
 - $\int \sin^3(x) dx$;
 - $\int \frac{dx}{x^2 + 1}$.
- (7 points) Show that the function

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}$$
 has no limit as (x, y) approaches $(0, 0)$.
- (7 points) Compute the double integral:

$$\int_0^1 \int_y^1 \frac{\sin(x)}{x} dx dy.$$