東吳大學 107 學年度暑假轉學生招生考試試題

系級	財務工程與精算數學系三年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

請詳述計算過程與理由，僅寫答案者不予計分

1. (10%) For what value of the constant c is the function g continuous on $(-\infty,\infty)$?

$$g(x)=\begin{cases}3x^2+cx, & if\ x<3,\\ cx^3-6x^2, & if\ x\geq 3.\end{cases}$$

2. (10%) Find the first derivative of the following function:

$$f(x)=\exp(\cos 5^x+\ln(\sin x^2))$$

3. (10%) Show that the geometric series

$$a+ar+ar^2+\cdots+ar^n+\cdots=\sum_{n=0}^{\infty}ar^n$$

converges if $|r|<1$ and derive its sum.

4. (10%) Show that

$$\lim_{(x,y)\to(0,0)}\frac{3x^2-5y^2}{x^2+2y^2}$$

does not exist.

5. (15%) Find the extreme values of f on the region described by the inequality:

$$f(x,y)=e^{-xy},\qquad x^2+4y^2\leq 1.$$

6. (10%) Find the area enclosed by the line $y=2x-1$ and the parabola $y^2=4x+6$.

7. (15%) Evaluate the integral

$$\int_0^{\infty}e^{-x^2}dx.$$

8. (10%) Evaluate the integral

$$\int_0^1\int_1^2(2x+e^{-y})dxdy.$$

9. (10%) Find the integral

$$\int\frac{xe^{2x}}{(1+2x)^2}dx.$$