東吳大學 107 學年度暑假轉學生招生考試試題

系級	巨量資料管理學院學士學位學程二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (10%) Find the limit if it exists

(a) $\lim_{x\to 0}\dfrac{e^x-\ln(1+x)-1}{x^2}$ **(b)** $\lim_{x\to 1}\left(\dfrac{1}{x-1}-\dfrac{x}{\ln x}\right)$

2. (10%) If z = f(x,y), where x= g(t), y=h(t), g(3)=2, g'(3)=5, h(3)=7, h'(3)=-4, f_x=(2,7)=6 and f_y(2,7)=-8, find $\dfrac{dz}{dt}$ when t=3.

3. (10%) Find the indefinite integral.

(a) $\int\dfrac{1-\sqrt{x}}{1+\sqrt{x}}dx$ **(b)** $\int x^3\left(x^2+1\right)^{3/2}dx$

4. (10%) Let $G(x)=\int_0^x\left[\int_0^s f(t)dt\right]ds$, **where** f **is continuous for all real** t**. Find(a)** $G(0)$ **(b)** $G'(0)$**.**

5. (10%) The Gamma Function $\Gamma(n)$ **is defined by**

$$\Gamma(n)=\int_0^\infty x^{n-1}e^{-x}dx \quad , \quad n>0.$$

Find $\Gamma(1)$**,** $\Gamma(2)$ **and** $\Gamma(3)$**.**

6. (10%) Find the local maximum and minimum values and saddle point(s) of the function.

$$f(x,y)=xy(1-x-y)$$

7. (10%) Find the absolute maximum and minimum values of f on the set D.

$$f(x,y)=x^2+y^2+x^2y+4, \qquad D=\{(x,y)\,|\,|x|\le 1,|y|\le 1\}$$

8. (10%) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s).

$$f(x,y,z)=yz+xy; \qquad xy=1, \qquad y^2+z^2=1$$

9. (10%) Evaluate the iterated integral.

$$\int_1^e\int_1^2\frac{(\ln y)^2}{x^2y}dxdy$$

10. (10%) Evaluate the double integral:

$f(x,y)=ye^{x^3}$ **; R is bounded by** $x=\frac{y}{2}, x=1, \text{and } y=0$