

東吳大學 108 學年度暑假轉學生招生考試試題

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系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (10%) Suppose that f is differentiable everywhere and $y = \sin(f(x^3))$. Find $\frac{dy}{dx}$.

2. (15%) Let $f(x, y) = \begin{cases} xy \left(\frac{x^2 - y^2}{x^2 + y^2} \right) & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

Find (a) $\frac{\partial f}{\partial y}(0, 0)$ (b) $\frac{\partial f}{\partial y}(x, y)$ for $(x, y) \neq (0, 0)$ (c) $\frac{\partial^2 f}{\partial x \partial y}(0, 0)$.

3. (10%) Evaluate the integral $\int_1^3 x^4 \ln x dx$.

4. (15%) Find what values of a and b in the following equation true?

$$\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0$$

5. (10%) If f is continuous and $\int_0^9 f(x) dx = 4$, find $\int_0^3 xf(x^2) dx$

6. (15%) Prove that

$$1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} < e^x$$

for every $x \neq 0$ and every $n \in \mathbb{N}$.

7. (10%) Let $S = \{(x, y) : y \geq 0, x^2 + y^2 \leq 1\}$. Find $\iint_S (x + 3y^3) dA$

8. (15%) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2 + 1}$$