東吳大學 108 學年度暑假轉學生招生考試試題

系級	數學系三年級	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

LINEAR ALGEBRA TRANSFERRING EXAM.

1. (20 points) Suppose $A = \begin{pmatrix} 1 & 5 & 8 \\ 0 & 1 & 1 \\ -2 & 3 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & 0 \\ -1 & -3 & -5 \\ 0 & 1 & 0 \end{pmatrix}$. Compute $A+B$, AB, $\operatorname{tr}(AB)$, and $\det(AB)$.

2. (10 points) Let A, B, C be matrices. If $AB = AC$, then is it true that $B = C$?

3. (15 points) Suppose that $A = \begin{pmatrix} 1 & -2 & 1 & 1 \\ -1 & 3 & 0 & 2 \\ 0 & 1 & 1 & 2 \end{pmatrix}$. Find the rank of A.

4. (15 points) In $\mathbf{R}^3$. Let $e_1 = (1,1,0)$ and $e_2 = (1,1,1)$. Find $e_3 \in \mathbf{R}^3$ such that $\{e_1, e_2, e_3\}$ form a basis of $\mathbf{R}^3$ over $\mathbf{R}$.

5. (15 points) Write the matrix presentation of the linear transformation:
$$T : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$$
sending (x,y,z) to $T(x,y,z) = (3x - y, y + 2z, x + y + z)$ with respect to the standard basis $e_1 = (1,0,0)$, $e_2 = (0,1,0)$, and $e_3 = (0,0,1)$.

6. (15 points) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation such that
$$T(1,1,1) = (2,2),\ T(0,1,1) = (0,1),\ T(0,0,1) = (-1,1).$$
Find the null space of T.

7. (10 points) Suppose A, B, C are matrices and $A = BCB^{-1}$ with $C = \begin{pmatrix} 3 & 7 & 9 \\ 0 & 1 & 1 \\ 0 & 0 & 5 \end{pmatrix}$. Find $\operatorname{tr}(A)$ and $\det(A)$.