東吳大學 108 學年度暑假轉學生招生考試試題

系級	財務工程與精算數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

1. (10 points) 是非題, 請依照順序標清楚題號以 O 或 X 作答:
   (a) (1%) By L'Hôpital's rule, $\lim_{x\to\infty} \frac{x+\sin x}{x}$ dose not exist.
   (b) (1%) $\int_{-1}^{1}\int_{-2}^{2} y\,dxdy = \int_{-1}^{1}\int_{-2}^{2} y\,dydx$.
   (c) (1%) $\frac{x^2-4}{x-2} = x+2$.
   (d) (1%) No point on the curve $x^2-3xy+y^2=1$ such that $\frac{dy}{dx}=0$.
   (e) (1%) If $f(x,y)$ has two local maxima, then f at least have one local minimum.
   (f) (1%) $\int_{-1}^{1} \frac{1}{x^2}dx = -2$
   (g) (1%) There exists a function f such that $f(x)<0$, $f'(x)<0$, and $f''(x)>0$ for all x.
   (h) (1%) The most general antiderivative of $f(x)=x^{-3}$ is $F(x) = -\frac{1}{2}x^{-2}+C$ for all x.
   (i) (1%) $(3^x)' = x\cdot 3^{x-1}$.
   (j) (1%) $\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$.

2. (30 points) 填充題, 不須寫出過程:
   (a) (5%) $\lim_{x\to 0} \frac{\sin x}{x} = ?$
   (b) (5%) $\lim_{x\to\infty} (\ln x - x) = ?$
   (c) (5%) If f is a differentiable function, $f(g(x)) = x$ and $f'(x) = 1+f(x)$, find $g'(x) = ?$
   (d) (5%) $\int \ln x\,dx = ?$
   (e) (5%) $\int_0^\infty e^{-\frac{x^2}{2}}dx = ?$
   (f) (5%) Find the 8th-degree Taylor polynomial of $\cos x$ centered at $x=0$.

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3. (60 points) 計算題, 請詳述計算過程與理由，僅寫答案者不予計分:
   (a) (10%) Find the tangent line of the function $f(x) = (\ln x)^{\cos x}$ at $x=e$.
   (b) (10%) Show that $\frac{d}{dx}(\sin x) = \cos x$.
   (c) (10%) Is the series $\sum_{n=1}^{\infty} (\frac{n}{n+1})^{n^2}$ convergent or divergent?
   (d) (10%) Prove or disprove that $\lim_{x\to 0} f(x) = 0$, if

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

   (e) (10%) Prove or disprove that $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2$, if $z=f(x,y)$, $x = r\cos\theta$ and $y = r\sin\theta$.
   (f) (10%) A rectangular box is resting on the $xy-$plane with one vertex at origin. The opposite vertex lie in the plane $2x+3y+5z=90$. Find the maximize the volume. (Hint: Use Lagrange multiplier method)