東吳大學109學年度碩士班研究生招生考試試題

系級	數學系碩士班	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

CALAULUS GRADUATE ENTRANCE EXAM

1. (5 points for each) Compute the derivatives $f'(x)$ of the following functions:
   (a) $f(x) = \ln(|x|)$.
   (b) $f(x) = (x^2+1)^7$.
   (c) $f(x) = \frac{x^2+1}{x}$.
   (d) $f(x) = e^{3x}$.
   (e) $f(x) = \frac{4}{3}\ln(|3x-1|)$.

2. (4 points for each)
   (a) We say that $x = a$ is a critical point of $f(x)$ if?
   (b) Find all critical points of $f(x) = x^3$.

3. (3 points for each) Find the absolute maximum and absolute minimum of
$$f(x) = x^3 - x$$
on the following domains:
   (a) $[-1, 1]$.
   (b) $[-2, 2]$.
   (c) $(-2, 2)$.
   (d) $(0, \infty)$.
   (e) $(-\infty, \infty)$.

4. (7 points) Suppose $f'(x) = xe^x$ and $f(0) = 1$, then what is $f(x)$?

5. (5 points for each) Compute the following definite integrals:
   (a) $\int_0^1 x^2 dx$.
   (b) $\int_0^\pi \cos(x) dx$.
   (c) $\int_{-1}^1 3x^2\sqrt{x^3+1}$.
   (d) $\int_0^1 \ln(x) dx$.
   (e) $\int_0^{\pi/2} \sin^3(x) dx$.

6. (5 points for each) Compute the following indefinite integrals:
   (a) $f(x) = \int \frac{1}{x} dx$.
   (b) $f(x) = \int 14x(x^2+1)^6 dx$.
   (c) $f(x) = \int \frac{x^2-1}{x^2} dx$.
   (d) $f(x) = \int 3e^{3x} dx$.