東吳大學 109 學年度碩士班研究生招生考試試題

系級	國際經營與貿易學系碩士班 A 組(國際貿易與金融)	考試時間	100 分鐘
科目	統計學	本科總分	100 分

注意事項：

一、 一律作答於答案卷上（題上作答不予計分），並務必標明題號，依序請使用中文作答。

二、 不可用手機與具有記憶功能之計算機或電子器具。

三、 所需參考資料請參見附表。

PART A: MUTIPLE CHOICE (20%) You don't have to show any work. Just select the best choice.

A1. (5%) 在一可裝 6 發子彈的左輪手槍裡，只放一顆子彈，隨機地一轉後，要求二位戰俘輪流用手槍向自己的頭部發射，直到一名戰俘中槍，另一名戰俘才逃過一劫。小明與小華聽完此情境後，各自發表了看法：

小明說：先發射者比較不利（意旨死亡機率較高）。

小華說：若改為放兩顆子彈，我認為先發者比較不利（意旨死亡機率較高）。

(A) 小明說對了，而且小華也說對了。

(B) 小明說對了，但小華說錯了。

(C) 小明說錯了，但小華說對了。

(D) 小明與小華都說錯了。

A2. (5%) 有甲、乙、丙三囚犯，國王宣佈以抽籤決定釋放其中一位，處決另兩位。國王抽籤完後告訴獄卒哪一位將被釋放，但要求獄卒不可先透露。甲請求獄卒透露哪一位會被釋放，可惜遭到拒絕，於是他改問獄卒『乙及丙中，哪一位會被處決？『獄卒經過一番思考，（誠實地）告訴甲，『乙會遭處決』。獄卒認為這樣做並未違反國王的規定。

(A) 甲聽完獄卒的說法後，他會被釋放的機率提高至 1/2。

(B) 甲聽完獄卒的說法後，他會被釋放的機率提高至 2/3。

(C) 甲聽完獄卒的說法後，他會被釋放的機率依舊不變，仍然是 1/3。

(D) 以上皆非。

A3. (5%) Consider the following statements about a 95% confidence interval (CI) for a parameter θ.

Statement 1: $P(\theta_0 \; is \; in \; the \; CI | \theta = \theta_0) \geq 0.95$

Statement 2: $P(\theta_0 \; is \; in \; the \; CI) \geq 0.95$

Statement 3: An experiment produced the $CI[-1, 1.5]$: $P(\theta \; is \; in \; [-1, 1.5] | \theta = 0) \geq 0.95$.

(A) Only Statement 1 is correct.

(B) Only Statement 2 is correct.

(C) Only Statement 3 is correct.

(D) Only Statement 1 is incorrect.

(E) Only Statement 2 is incorrect.

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A4. (5%) Consider the linear regression model: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_m x_m + \varepsilon$. The data set is $\{y_i, x_{i1}, x_{i2}, \ldots, x_{im}\}_{i=1}^{n}$. After fitting these data into this model, you obtained the linear system which is denoted by $Y = X\beta + \varepsilon$. Which of the followings is <u>NOT TRUE</u>?

(A) $X = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \ldots & x_{nm} \end{bmatrix}$

(B) If $x_2 = x_1^2$, then we cannot find the least square estimator for β.

(C) Follow the least square criterion, we have the residuals. The sum of residuals must be zero.

(D) If you add new independent variable x_{m+1} into the original linear regression model, *i.e.* $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_m x_m + \beta_{m+1} x_{m+1} + \varepsilon$, then the coefficient of determination won't decrease.

PART B: QEUSTIONS (30%) Please provide your explanations by sentences, formulas, or figures.

B1. (5%) What is the *central limit theorem*?
B2. (5%) What is a *consistent estimator*?
B3. (5%) What is a *type II error*?
B4. (5%) What is the purpose to calculate the *adjusted coefficient of determination*?
B5. (5%) What is the main difference between *linear regression* and *logistic regression*?
B6. (5%) What is the purpose to conduct the *Kolmogorov-Smirnov test*?

PART C: PROBLEMS (50%) Please provide your solutions in detail.

C1. (15%) A political poll is taken to determine the fraction p of the population that would support a referendum requiring all citizens to be fluent in the language of probability and statistics.

(1) **(7%)** Assume $p = 0.5$. Use the central limit theorem to estimate the probability that in a poll of 25 people, at least 14 people support the referendum. Your answer to this problem should be a decimal (up to thousandths place).

(2) **(8%)** With p unknown and n the number of random people polled, let $\overline{X_n}$ be the fraction of the polled people who support the referendum. What is the smallest sample size n in order to have a 90% confidence that $\overline{X_n}$ is within 0.01 of the true value of p? Your answer to this problem should be an integer.

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C2. (15%) Someone claims to have found a long-lost work by Jane Austen. She asks you to decide whether or not the book was actually written by Austen. You buy a copy of *Sense and Sensibility* and count the frequencies of certain common words on some randomly selected pages. You do the same thing for the 'long-lost work'. You get the following table of counts.

Word	a	an	this	that
Sense and Sensibility	150	30	30	90
Long-lost work	90	20	10	80

Using this data, set up and evaluate a significance test of the claim that the long-lost book is by Jane Austen. Use a significance level of 0.1.

C3. (20%) Given data $\{(x_i, y_i)\}_{i=1}^{4}$ and they are (0, 1) (4, 3) (4, 7) (0, 1), respectively. You use two linear regression models (called $Model\ 1$ and $Model\ 2$) with two parameters c and d to fit these data points. They are state as follows:

$$Model\ 1: y = c + dx + \varepsilon$$
$$Model\ 2: y = c + d\sqrt{x} + \varepsilon$$

(1) **(4%)** Find the least square estimator for c and d in the $Model\ 1$.
(2) **(4%)** Find the least square estimator for c and d in the $Model\ 2$.
(3) **(4%)** Find SSE_{m1} (sum of squared residuals of the $Model\ 1$).
(4) **(4%)** Find SSE_{m2} (sum of squared residuals of the $Model\ 2$).
(5) **(2%)** Find R^2_{m1} and R^2_{m2} (coefficient of determination in $Model\ 1$ and in $Model\ 2$ respectively).
(6) **(2%)** According to the value of R^2, which Model ($Model\ 1$ or $Model\ 2$) is better?

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APPENDIX 1

Standard normal table of left tail probabilities.

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
-4.00	0.0000	-2.00	0.0228	0.00	0.5000	2.00	0.9772
-3.95	0.0000	-1.95	0.0256	0.05	0.5199	2.05	0.9798
-3.90	0.0000	-1.90	0.0287	0.10	0.5398	2.10	0.9821
-3.85	0.0001	-1.85	0.0322	0.15	0.5596	2.15	0.9842
-3.80	0.0001	-1.80	0.0359	0.20	0.5793	2.20	0.9861
-3.75	0.0001	-1.75	0.0401	0.25	0.5987	2.25	0.9878
-3.70	0.0001	-1.70	0.0446	0.30	0.6179	2.30	0.9893
-3.65	0.0001	-1.65	0.0495	0.35	0.6368	2.35	0.9906
-3.60	0.0002	-1.60	0.0548	0.40	0.6554	2.40	0.9918
-3.55	0.0002	-1.55	0.0606	0.45	0.6736	2.45	0.9929
-3.50	0.0002	-1.50	0.0668	0.50	0.6915	2.50	0.9938
-3.45	0.0003	-1.45	0.0735	0.55	0.7088	2.55	0.9946
-3.40	0.0003	-1.40	0.0808	0.60	0.7257	2.60	0.9953
-3.35	0.0004	-1.35	0.0885	0.65	0.7422	2.65	0.9960
-3.30	0.0005	-1.30	0.0968	0.70	0.7580	2.70	0.9965
-3.25	0.0006	-1.25	0.1056	0.75	0.7734	2.75	0.9970
-3.20	0.0007	-1.20	0.1151	0.80	0.7881	2.80	0.9974
-3.15	0.0008	-1.15	0.1251	0.85	0.8023	2.85	0.9978
-3.10	0.0010	-1.10	0.1357	0.90	0.8159	2.90	0.9981
-3.05	0.0011	-1.05	0.1469	0.95	0.8289	2.95	0.9984
-3.00	0.0013	-1.00	0.1587	1.00	0.8413	3.00	0.9987
-2.95	0.0016	-0.95	0.1711	1.05	0.8531	3.05	0.9989
-2.90	0.0019	-0.90	0.1841	1.10	0.8643	3.10	0.9990
-2.85	0.0022	-0.85	0.1977	1.15	0.8749	3.15	0.9992
-2.80	0.0026	-0.80	0.2119	1.20	0.8849	3.20	0.9993
-2.75	0.0030	-0.75	0.2266	1.25	0.8944	3.25	0.9994
-2.70	0.0035	-0.70	0.2420	1.30	0.9032	3.30	0.9995
-2.65	0.0040	-0.65	0.2578	1.35	0.9115	3.35	0.9996
-2.60	0.0047	-0.60	0.2743	1.40	0.9192	3.40	0.9997
-2.55	0.0054	-0.55	0.2912	1.45	0.9265	3.45	0.9997
-2.50	0.0062	-0.50	0.3085	1.50	0.9332	3.50	0.9998
-2.45	0.0071	-0.45	0.3264	1.55	0.9394	3.55	0.9998
-2.40	0.0082	-0.40	0.3446	1.60	0.9452	3.60	0.9998
-2.35	0.0094	-0.35	0.3632	1.65	0.9505	3.65	0.9999
-2.30	0.0107	-0.30	0.3821	1.70	0.9554	3.70	0.9999
-2.25	0.0122	-0.25	0.4013	1.75	0.9599	3.75	0.9999
-2.20	0.0139	-0.20	0.4207	1.80	0.9641	3.80	0.9999
-2.15	0.0158	-0.15	0.4404	1.85	0.9678	3.85	0.9999
-2.10	0.0179	-0.10	0.4602	1.90	0.9713	3.90	1.0000
-2.05	0.0202	-0.05	0.4801	1.95	0.9744	3.95	1.0000

$\Phi(z) = P(Z \leq z)$ for $N(0,1)$.

(Use interpolation to estimate z values to a 3rd decimal place.)

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APPENDIX 2

Table of χ^2 critical values (right-tail)

The table shows $c_{df,p}$ = the $1 - p$ quantile of $\chi^2(df)$.

In R notation $c_{df,p}$ = `qchisq(1-p, df)`.

df\p	0.010	0.025	0.050	0.100	0.200	0.300	0.500	0.700	0.800	0.900	0.950	0.975	0.990
1	6.63	5.02	3.84	2.71	1.64	1.07	0.45	0.15	0.06	0.02	0.00	0.00	0.00
2	9.21	7.38	5.99	4.61	3.22	2.41	1.39	0.71	0.45	0.21	0.10	0.05	0.02
3	11.34	9.35	7.81	6.25	4.64	3.66	2.37	1.42	1.01	0.58	0.35	0.22	0.11
4	13.28	11.14	9.49	7.78	5.99	4.88	3.36	2.19	1.65	1.06	0.71	0.48	0.30
5	15.09	12.83	11.07	9.24	7.29	6.06	4.35	3.00	2.34	1.61	1.15	0.83	0.55
6	16.81	14.45	12.59	10.64	8.56	7.23	5.35	3.83	3.07	2.20	1.64	1.24	0.87
7	18.48	16.01	14.07	12.02	9.80	8.38	6.35	4.67	3.82	2.83	2.17	1.69	1.24
8	20.09	17.53	15.51	13.36	11.03	9.52	7.34	5.53	4.59	3.49	2.73	2.18	1.65
9	21.67	19.02	16.92	14.68	12.24	10.66	8.34	6.39	5.38	4.17	3.33	2.70	2.09
10	23.21	20.48	18.31	15.99	13.44	11.78	9.34	7.27	6.18	4.87	3.94	3.25	2.56
16	32.00	28.85	26.30	23.54	20.47	18.42	15.34	12.62	11.15	9.31	7.96	6.91	5.81
17	33.41	30.19	27.59	24.77	21.61	19.51	16.34	13.53	12.00	10.09	8.67	7.56	6.41
18	34.81	31.53	28.87	25.99	22.76	20.60	17.34	14.44	12.86	10.86	9.39	8.23	7.01
19	36.19	32.85	30.14	27.20	23.90	21.69	18.34	15.35	13.72	11.65	10.12	8.91	7.63
20	37.57	34.17	31.41	28.41	25.04	22.77	19.34	16.27	14.58	12.44	10.85	9.59	8.26
21	38.93	35.48	32.67	29.62	26.17	23.86	20.34	17.18	15.44	13.24	11.59	10.28	8.90
22	40.29	36.78	33.92	30.81	27.30	24.94	21.34	18.10	16.31	14.04	12.34	10.98	9.54
23	41.64	38.08	35.17	32.01	28.43	26.02	22.34	19.02	17.19	14.85	13.09	11.69	10.20
24	42.98	39.36	36.42	33.20	29.55	27.10	23.34	19.94	18.06	15.66	13.85	12.40	10.86
25	44.31	40.65	37.65	34.38	30.68	28.17	24.34	20.87	18.94	16.47	14.61	13.12	11.52
30	50.89	46.98	43.77	40.26	36.25	33.53	29.34	25.51	23.36	20.60	18.49	16.79	14.95
31	52.19	48.23	44.99	41.42	37.36	34.60	30.34	26.44	24.26	21.43	19.28	17.54	15.66
32	53.49	49.48	46.19	42.58	38.47	35.66	31.34	27.37	25.15	22.27	20.07	18.29	16.36
33	54.78	50.73	47.40	43.75	39.57	36.73	32.34	28.31	26.04	23.11	20.87	19.05	17.07
34	56.06	51.97	48.60	44.90	40.68	37.80	33.34	29.24	26.94	23.95	21.66	19.81	17.79
35	57.34	53.20	49.80	46.06	41.78	38.86	34.34	30.18	27.84	24.80	22.47	20.57	18.51
40	63.69	59.34	55.76	51.81	47.27	44.16	39.34	34.87	32.34	29.05	26.51	24.43	22.16
41	64.95	60.56	56.94	52.95	48.36	45.22	40.34	35.81	33.25	29.91	27.33	25.21	22.91
42	66.21	61.78	58.12	54.09	49.46	46.28	41.34	36.75	34.16	30.77	28.14	26.00	23.65
43	67.46	62.99	59.30	55.23	50.55	47.34	42.34	37.70	35.07	31.63	28.96	26.79	24.40
44	68.71	64.20	60.48	56.37	51.64	48.40	43.34	38.64	35.97	32.49	29.79	27.57	25.15
45	69.96	65.41	61.66	57.51	52.73	49.45	44.34	39.58	36.88	33.35	30.61	28.37	25.90
46	71.20	66.62	62.83	58.64	53.82	50.51	45.34	40.53	37.80	34.22	31.44	29.16	26.66
47	72.44	67.82	64.00	59.77	54.91	51.56	46.34	41.47	38.71	35.08	32.27	29.96	27.42
48	73.68	69.02	65.17	60.91	55.99	52.62	47.34	42.42	39.62	35.95	33.10	30.75	28.18
49	74.92	70.22	66.34	62.04	57.08	53.67	48.33	43.37	40.53	36.82	33.93	31.55	28.94