

東吳大學 109 學年度暑假轉學生招生考試試題

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系級	數學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. (5 points for each) Evaluate the following indefinite integrals:
 - (a) $\int (x^3 + 1)dx.$
 - (b) $\int \sin^3(x)dx.$
 - (c) $\int \ln(x)dx.$
 - (d) $\int xe^x dx.$

2. (5 points for each) Determine the convergence or divergence of the following series:
 - (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.2}}.$
 - (b) $\sum_{n=1}^{\infty} \frac{n-1}{100n}.$
 - (c) $\sum_{n=1}^{\infty} \frac{4^n}{5^n}.$
 - (d) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^n}\right).$

3. (8 points) Write down the Taylor series of $\cos(x)$ about $x = 0$.

4. (6 points for each) Compute the following limits:
 - (a) $\lim_{(x,y) \rightarrow (1,1)} \frac{xy-y-2x+2}{x-1}.$
 - (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x-y}{x+y}.$

5. (6 points for each) Compute f_{xx}, f_{xy}, f_{yy} of the following functions:
 - (a) $f(x, y) = x \sin(xy).$
 - (b) $f(x, y) = e^{(x+y+1)}.$

6. (6 points) Suppose $f(x, y, z) = x^3y^4z^5$ and $x = \sin(t), y = e^t, z = t^2$. Please compute

$$\frac{df(x, y, z)}{dt}.$$

7. (6 points) Find the directional derivative of $f(x, y) = 2xy - 3y^2$ at the point $(x, y) = (5, 5)$ and in the direction $\mathbf{u} = (4, -3)$.

8. (8 points) Find all the local maximal, local minimal, and saddle points of the function

$$f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy.$$

9. (8 points) Find the point $P = (x, y, z)$ on the plane $3x - y + 5z - 1 = 0$ that is closest to $(0, 0, 0)$.