

# 東吳大學 109 學年度暑假轉學生招生考試試題

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系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

一、(16%) Let  $\lim x_n = a$ . Prove that  $\{x_n\}$  is Cauchy and bounded.

二、(12 %) Find the limit if it exists.

(1)  $\lim_{x \rightarrow 0} \tan x \sin\left(\frac{1}{x^2}\right)$ ,      (2)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x-4} - \sqrt{x+1}}{\sqrt{x-3} - \sqrt{x+1}}$

三、(10%) Show that  $\cos x \leq e^x$  for all  $x \geq 0$

四、(12%) Show that  $f(x) = \frac{\sin x}{x}$  is uniformly continuous on  $(0, 1000)$ .

五、(20 %) Let  $f : [a, b] \rightarrow \mathbb{R}$ .

(1) What is  $f$  integrable on  $[a, b]$ ? 5%

(2) Prove that if  $f$  integrable on  $[0, 1]$ , then  $\lim_{n \rightarrow \infty} \sqrt[n]{n} \int_0^1 f(x) dx = 0$  10%

(3) Suppose that  $f : \mathbb{R} \rightarrow \mathbb{R}$  is continuous and  $F(x) = \int_0^{x \cos x} f(t) dt$ . Find  $F'(x)$ . 5%

六、(16 %) Prove that first-order partial derivatives of  $f(x, y) = \begin{cases} x+y & x=0 \text{ or } y=0 \\ 1 & \text{otherwise} \end{cases}$

exist at  $(0, 0)$ , but  $f$  is not continuous at  $(0, 0)$

七、(14 %) Suppose that  $V$  is open and connected in  $\mathbb{R}^n$ , and that  $\vec{f} : V \rightarrow \mathbb{R}^m$  is differentiable on  $V$  such that  $D\vec{f}(\vec{c}) = \vec{0}$  for all  $\vec{c} \in V$ . Prove that  $\vec{f}$  is constant in  $V$ .