

東吳大學 110 學年度碩士班研究生招生考試試題

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系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

CALAULUS GRADUATE ENTRANCE EXAM.

1. (10 points) Let $f(x)$ be a function on $[a, b]$. Show that if $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on $[a, b]$.
2. (10 points) Find absolute maximal/minimal of $f(x) = 2 \sin(x) - \cos(2x)$ in $[0, 2\pi]$.
3. (10 points) Find absolute maximal/minimal of $f(x) = x^3 - 12x - 5$ in $(0, \infty)$.
4. (10 points) Sketch the graph of $f(x) = x^4 - 4x^3$.
5. (7 points for each) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x + x^2}$.
6. (10 points) Find the shortest distance from the point $(x, y) = (0, 2)$ to the parabola $y = 4 - x^2$.
7. (8 points) Let $f(x)$ be a function. What is n -th Riemann sum $R_n(f)$ on $[0, 1]$?
8. (35 points) Evaluate the following integrals:
 - (a) $\int_0^\pi \cos(x) dx$;
 - (b) $\int_1^4 \left(\frac{3}{2} \sqrt{x} - \frac{4}{x^2} \right) dx$;
 - (c) $\int \frac{1}{x} dx$;
 - (d) $\int \tan(x) dx$;
 - (e) $\int \frac{1}{x^2+1} dx$;
 - (f) $\int x \sqrt{2x+1} dx$;
 - (g) $\int \sin(x) e^{\cos(x)} dx$.