

# 東吳大學 110 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. 20%

- (a) In  $\mathbf{R}^3$ , represent all the linear combinations of  $\mathbf{v}=(2,0,1)$  and  $\mathbf{w}=(0,1,2)$ , give a geometric interpretation. And find a vector which is not a combination of  $\mathbf{v}$  and  $\mathbf{w}$ .
- (b) In the following two planes, which one is a subspace of  $\mathbf{R}^3$  (give your reasons) and find a basis for the subspace.  $x + 2y - 3z - 4 = 0$ ,  $2x - 4y + 5z = 0$

2. 20%

- (a) 6% If  $\mathbf{A}$  is a  $3 \times 3$  matrix has the determinant  $\det(\mathbf{A}) = -5$ , then what is  $\det(3\mathbf{A}^{-1})$  and  $\det(\mathbf{A}^T)$  ?
- (b) 6% Let  $\mathbf{E}$  be an  $n \times n$  elementary matrix, what is the determinant  $\det(\mathbf{E})$  of  $\mathbf{E}$  ?

- (c) 8% For the matrix  $\mathbf{A} = [a_{ij}]_{3 \times 3} = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix}$ , find the value of  $a_{11}C_{31} + a_{12}C_{32} + a_{13}C_{33}$ , where  $C_{ij}$  is the

cofactor of entry  $a_{ij}$ .

3. 20% For  $\mathbf{A} = \begin{bmatrix} 2 & 4 & 2 & 10 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 3 & 1 \end{bmatrix}$ , find the reduced row echelon form of  $\mathbf{A}$  and verify the dimension

theorem for matrices.

4. 20% Find the eigenvalues and eigenvectors of the matrix  $\mathbf{A} = \begin{bmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{bmatrix}$ . Is  $\mathbf{A}$  diagonalizable? And

find the Jordan form of  $\mathbf{A}$ .

5. 20% Let  $\mathbf{R}^3$  have the Euclidean inner product, and let  $\mathbf{v}_1 = (1, 0, 0)$ ,  $\mathbf{v}_2 = (1, 0, -1)$ ,  $\mathbf{v}_3 = (1, 1, 0)$ . Use the Gram-Schmidt process to find the QR-decomposition of  $\mathbf{A} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$ .