

東吳大學 110 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 B 組(決策科學與海量資料分析)	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. (8 points) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 + 2x^2 - 1}{5 - 3x}$.
2. (8 points) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$.
3. (8 points) Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{7x^2 + 6}}{3x - 5}$.
4. (8 points) Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$.
5. (8 points) Let $y = \sin x - \cos x$. Find $\frac{d^{91}y}{dx^{91}}$.
6. (8 points) Let $f(x) = xg(x)$, $g(3) = 5$, and $g'(3) = 2$. Find $f'(3)$.
7. (8 points) Let $f'(x) = x\sqrt{x}$ and $f(1) = 2$. Find $f(x)$.
8. (8 points) Evaluate $\int_{-1}^3 \frac{1}{x^2} dx$.
9. (8 points) Evaluate $\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx$.
10. (8 points) Find $\int x^5 \sqrt{1 + x^2} dx$.
11. (10 points) Let $f'(x)$ exist everywhere. If $f(2) = 8$ and $f'(x) \leq 5$ for all x . Use the mean value theorem to answer how small can $f(0)$ possibly be?
12. (10 points) Let $f(x) = \int_0^x \sqrt{1 + t^2} dt$, $x \geq 0$. Use the mean value theorem for definite integral and the limit-definition of derivative to find $f'(x)$.

(Mean Value Theorem) Suppose $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b) . Then there exists $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

(Mean Value Theorem for Definite Integral) Suppose $f : [a, b] \rightarrow \mathbb{R}$ is a continuous function. Then there exists $c \in [a, b]$ such that $\int_a^b f(t) dt = f(c)(b - a)$.