東吳大學 111 學年度碩士班研究生招生考試試題

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. (20%) Consider the homogeneous linear system

$$\begin{array}{rcrcrcrcrcrcl} x_1 & + & 3x_2 & - & 2x_3 & & & + & 2x_5 & & & = & 0 \\ 2x_1 & + & 6x_2 & - & 5x_3 & - & 2x_4 & + & 4x_5 & - & 3x_6 & = & 0 \\ & & & & 5x_3 & + & 10x_4 & & & + & 15x_6 & = & 0 \\ 2x_1 & + & 6x_2 & & & + & 8x_4 & + & 4x_5 & + & 18x_6 & = & 0 \end{array}$$

Using Gaussian elimination, find the general solution of the linear system, and find a basis for the solution space.

2. (20%) Let n be a positive integer, and H_n be the set of all $n \times n$ symmetric matrices. Prove that H_n is a subspace of the space of $n \times n$ matrices. Find a basis for H_3. What is the dimension of H_3?

3. (20%) Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. Find an orthonormal matrix U so that $U^T AU$ is a diagonal matrix.

4. (20%) Let α and β be two positive real numbers, and let $A = \begin{pmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{pmatrix}$. Give a necessary and sufficient on α, β so that A is positive definite.

5. (20%) State the following theorems.

(5.1) Cauchy-Schwarz inequality.

(5.2) Dimension theorem of a matrix.

(5.3) Spectral decomposition theorem of a symmetric matrix.

(5.4) Cayley Hamilton theorem.