東吳大學 111 學年度碩士班研究生招生考試試題

系級	數學系碩士班	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. Evaluate the following limits.

   a. (10 points) $\lim_{x\to\infty}\sqrt{x^2+x}-x$.

   b. (10 points) $\lim_{x\to 0}\dfrac{x-\sin x}{x^3}$.

2. Evaluate the indefinite or definite integrals.

   a. (10 points) $\int \sec^3 x dx$.

   b. (10 points) $\int_{-2}^{2}\left(\dfrac{1}{2}+x^3\cos\dfrac{x}{2}\right)\sqrt{4-x^2}dx$.

3. (10 points) Find all the inflection points of $f(x)=\dfrac{x}{1+x^2}$.

4. (10 points) Find the tangent line to the curve $x\sin 2y = y\cos 2x$ at the point $\left(\dfrac{\pi}{4},\dfrac{\pi}{2}\right)$.

5. (10 points) Find $\dfrac{d}{dx}\int_{x^2}^{e^x}\sin(t^2)dt$.

6. Prove that the equation $x^3+x-1=0$ has exactly one real root on the interval $(0,1)$ by the following two steps.

   a. (5 points) Show the existence. You may use the Intermediate Value Theorem.

   b. (5 points) Show the uniqueness. You may use the Mean Value Theorem.

7. Consider the two functions $f:A\to B$ and $g:B\to C$. Assume that their composition $g\circ$ is a one-to-one function.

   a. (5 points) Is f necessarily one-to-one? Give your reasons.

   b. (5 points) Is g necessarily one-to-one? Give your reasons.

8. For each of the following statements, determine whether it is true or false. If it is true, give your reasons. If not, give a counterexample.

   a. (5 points) If f is a continuous function, then $|f|$ is a continuous function.

   b. (5 points) If $|f|$ is a continuous function, then f is a continuous function.