

東吳大學 105 學年度碩士班研究生招生考試試題

第 1 頁，共 1 頁

系級	數學系碩士班 A 組(數學)	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. (20 points) Using Gaussian elimination, find the general solution of the linear system

$$\begin{aligned} x - y - z + 2w &= 1 \\ 2x - 2y - z + 3w &= 3 \\ -x + y - z &= -3 \end{aligned}$$

2. (20 points) Let  $V = \{(x, y, z) \in \mathbb{R}^3 : 2x + 3y - 5z = 0\}$ . Prove that  $V$  is a subspace. Find a matrix  $A$  such that the null space of  $A$  is  $V$ .

3. (20 points) Let  $A, B$  be  $3 \times 3$  matrices. Suppose  $\det(A) = -5$  and  $\det(B) = 0$ . Find  $\det(2A)$ ,  $\det(A^3)$ ,  $\det(B^9A^2)$ ,  $\det(A^T A^{-1})$ .

4. (20 points) Prove that the vectors  $(1, 0, 1), (0, 1, 2), (2, 1, 0)$  in  $\mathbb{R}^3$  are linearly independent. Apply the Gram-Schmidt process to transform these three vectors into orthonormal vectors.

5. (20 points) Let  $A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ . Find an invertible matrix  $S$  and a diagonal matrix  $\Lambda$  such that  $S^{-1}AS = \Lambda$ . Apply this formula to compute  $A^{37}$ .