

系級	巨量資料管理學院學士學位學程二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

每題十分：

1. Let  $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } x > 1 \end{cases}$ . Find the values of  $a$  and  $b$  so that  $f$  is continuous and has a derivative at  $x = 1$ .

2. Evaluate the limit, using L'Hopital's rule if necessary

(a)  $\lim_{x \rightarrow 0^+} \frac{e^x - (1+x)}{x^n}$ , if  $n$  is a positive integer

(b)  $\lim_{x \rightarrow \infty} \frac{(\ln x)^n}{x^m}$ , where  $n, m$  is a positive integer, respectively.

3. Find  $\frac{dy}{dx}$ . (a)  $y = x^{x-1}$  (b)  $y = (1+x)^{1/x}$

4. Find the antiderivative or evaluate the integrals.

(1)  $\int \frac{x^2 + 1}{x^3 - x} dx$       (2)  $\int \frac{dx}{e^x - e^{-x}}$

5. Find the function  $f$  given that the slope of the tangent line to the graph of  $f$  at any point  $(x, f(x))$  is  $f'(x) = \frac{\ln x}{\sqrt{x}}$  and that the graph of  $f$  passes through the point  $(1, -2)$ .

6. Find the maximum and minimum values of the function  $f(x, y) = 2x - 3y + 1$  subject to the constraint  $2x^2 + 3y^2 - 125 = 0$ .

7. Find the solution of the initial-value problem.  $y' = \frac{xy}{x^2 + 1}; y(0) = 1$ .

8. Find the critical point(s) of the functions. Then use the second derivative test to classify the nature of each of these points, if possible. Finally, determine the relative extrema of the function.

$$f(x, y) = \ln(x^2 + y^2 - 2x - 2y + 4)$$

東吳大學 105 學年度轉學生(含進修學士班轉學生)招生考試試題

第 2 頁，共 2 頁

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9. Find the absolute extrema of the function over the region  $R$ .

$$f(x, y) = x^2 + xy \quad , \quad R = \{(x, y) : |x| \leq 2, |y| \leq 1\}$$

10. Find the average value of the function  $f(x, y)$  over the plane region  $R$ .

$$f(x, y) = xe^y ; R \text{ is the triangle with vertices } (0,0) , (1,0) \text{ and } (1,1).$$