

東吳大學 105 學年度轉學生(含進修學士班轉學生)招生考試試題

第 1 頁, 共 1 頁

系級	數學系三年級	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

1. 20%

- (a) Represent the following system of equations in matrix form  $Ax=b$ . Determine whether the system is consistent by the Gaussian Elimination, and find the solutions if your answer is yes.

$$\begin{aligned} 4x + 4y + 4z &= 4 \\ -2x - 3y - z &= 0 \\ 3x + 2z &= 1 \end{aligned}$$

- (b) Find the reduced row echelon form of  $A$ .  
 (c) Find the inverse of  $A$ , if it is invertible.

2. 20%

Let  $A = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$ , find all the eigenvectors of  $A$ . Is  $A$  diagonalizable? Find  $A^k$  and  $A^\infty = \lim_{k \rightarrow \infty} A^k$ .

3. 20% Let  $V=W=\mathbf{R}^2$ ,  $v=(v_1, v_2) \in V$ ,  $T$  is a linear transformation from  $V$  to  $W$ .

- (a) Suppose  $T$  transforms  $(1,2)$  to  $(3,5)$  and  $(2,0)$  to  $(4,1)$ . Find  $T(v)$ ,  $T^2(v)$  for  $v=(1, -2)$ .  
 (b) Suppose  $T(v) = v$  with standard basis for  $V$ , and basis vectors  $w_1=(2, 3)$  and  $w_2=(-1, 1)$  for  $W$ . Find a matrix  $A$  for  $T$ .

4. 20% For  $A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 4 & 2 & 0 \\ -1 & -3 & 0 & 5 \end{bmatrix}$ ,

- (a) Find a basis for the row space of  $A$ .  
 (b) Find a basis for the null space of  $A$ .  
 (c) Verify the dimension Theorem for matrices.

5. 20% Let  $\mathbf{R}^3$  have the Euclidean inner product. And let  $v_1 = (1, 0, -1)$ ,  $v_2 = (1, 1, 0)$ ,  $v_3 = (2, 1, -1)$ ,  $v_4 = (1, 0, 0)$ .

- (a) Find a basis for  $\mathbf{R}^3$  from the set  $\{v_1, v_2, v_3, v_4\}$ .  
 (b) Use the Gram-Schmidt process to transform the basis in (a) into an orthonormal basis.