東吳大學 107 學年度暑假轉學生招生考試試題

系級	數學系三年級	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1.（20%）Assume that $\sin x$ is continuous on R, prove that the function

$$f(x)=\begin{cases}\sin\dfrac{1}{x} & x\neq 0\\ 1 & x=0\end{cases}$$

is continuous on $(-\infty,0)$ and $(0,\infty)$, discontinuous at 0, and neither $f(0+)$ nor $f(0-)$ exists.

2.（20%）(i) State Mean Value Theorem.

(ii) Prove that the function $f(x)=\sin x$ is uniformly continuous on □ .

3.（20%）(i) Let $u=F(x+g(y))$. Find u_x and u_{xy}.

(ii) Find all the critical points of the function $f(x,y)=x^2+3y^4+4y^3-12y^2$ and tell whether it is a local maximum, local minimum, or a saddle point.

4.（20%）(i) Let f be integrable on $[a,b]$. For $x\in[a,b]$, let $F(x)=\int_a^x f(t)dt$. Prove that $F'(x)=f(x)$ whenever f is continuous at x.

(ii) Given $F(x)=\int_{\psi(x)}^{\varphi(x)} f(x,t)dt$, find $F'(x)$, assuming suitable smoothness conditions on φ,ψ and f.

5. (20%) Let $g_k(x)=(k^2x^2+1)^{-1}$ and $\delta>0$. Prove the following:

(i) $g_k\to g$ pointwise on □ , where $g(x)=\begin{cases}0 & x\neq 0\\ 1 & x=0\end{cases}$

(ii) $\{g_k\}$ does not converge uniformly to its limit on □ .

(iii) $\{g_k\}$ converges uniformly to its limit on $[\delta,\infty)$