東吳大學 114 學年度碩士班招生考試試題

系級	數學系碩士班(統計與資料分析)	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. (15%) Estimate the following limits.
   (i) $\lim_{x\to 1} \frac{\sqrt{2x^2+3x-4}-x}{x-1}$.
   (ii) $\lim_{x\to 0} \frac{x^2}{e^x-1-x}$.
   (iii) $\lim_{x\to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$.

2. (16 %) Compute the derivatives $\frac{d}{dx}$ or partial derivatives $\frac{\partial}{\partial x}$ of the following functions.
   (i) $\cos(x^2) + (\cos x)^2$.
   (ii) $(\ln x)^x$.
   (iii) $\sin(xy) + x^2 e^{xy}$.
   (iv) $x \tan^{-1}(xy)$.

3. (12%) Evaluate the definite integrals.
   (i) $\int_1^2 x \ln x \, dx$.
   (ii) $\int_0^1 \frac{1}{\sqrt{1+x^2}} \, dx$.

4. (12%) Assume that
$$f(x) = \int_1^x \sqrt{t^3-1}\, dt \quad \text{and} \quad g(x) = \int_{-\sqrt[3]{x}}^{\sqrt{x}} \sin(t^3)\, dt.$$
   (i) (8%) Compute $\frac{d}{dx} f(x)$ and $\frac{d}{dx} g(x)$.
   (ii) (4%) Find the length of the curve $y = f(x), 1 \le x \le 4$.

5. (22%)
   (i) (4%) Write down the Taylor expansion of $f(x)$ at 0.
   (ii) Let $f(x) = xe^x$.
      (a) (7%) Compute $\int xe^x \, dx$.
      (b) (4%) Expand $f(x)$ in a power series at 0.
      (c) (7%) Use (a), (b) to evaluate the value $\sum_{n=1}^{\infty} \frac{1}{n!(n+2)}$.

6. (7%) Compute the equation of the tangent line to the curve
$$x^2(x^2+y^2) = y^2$$
   at the point $(\sqrt{2}/2, \sqrt{2}/2)$.

7. (16%) Evaluate the double integrals.
   (i) $\iint_{[0,1]\times[-1,2]} (x^2+xy)\, dA(x,y)$.
   (ii) $\iint_{[1,2]\times[1,3]} \frac{x}{(x^2+y^2)^2}\, dA(x,y)$.