

東吳大學 114 學年度暑假轉學生招生考試試題

第1頁，共1頁

系級	經濟學系二年級	考試時間	100 分鐘
科目	微積分	本科總分	100 分

※一律作答於答案卷上(題上作答不予計分)；並務必標明題號，依序作答。

1. Find $f'(x)$ for each $f(x)$.

(a) $f(x) = x^\pi - e^x + \ln x - 2025 + \frac{7}{\sqrt[3]{x^3}}$ (5%)

(b) $f(x) = \frac{x^2 - 1}{e^x + 1}$ (5%)

(c) $f(x) = \sin(\cos(e^{3x^2+7}))$ (5%)

2. Let $f(x) = \ln(1 + x^2)$. Find $f^{(2024)}(0)$. (10%)

3. Compute the following integrals.

(a) $\int x^2 \cdot \sqrt{3 - x^3} dx$ (5%) (b) $\int_0^1 \frac{6x^2 - x + 2}{3x + 1} dx$ (5%) (c) $\int_0^{\frac{\pi}{2}} \sin^3(2x) \cdot \cos^4(2x) dx$ (5%)

4. Find $\int_0^\infty \sin x \cdot e^{-3x} dx$. (10%)

5. Evaluate the following double integrals.

(a) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{\sqrt{4-x^2-y^2}} dy dx$ (10%)

(b) $\int_0^2 \int_{x^2}^4 \sqrt{y} \cdot \sin y dy dx$ (10%)

6. Find the absolute maximum value and minimum values of

$$f(x, y) = x^2 + xy - 2y^2$$

subject to the constraint $2x^2 + 2xy + 5y^2 = 4$. (10%)

7. Evaluate the following limits.

(a) $\lim_{x \rightarrow 0} [2 - \cos(x^2)]^{\frac{\sin^3 x}{x^3 \cdot (1 - \cos(x^2))}}$ (5%)

(b) $\lim_{x \rightarrow 0} \frac{\int_0^x \tan^{-1}(t^2) dt}{\sin x + \cos x - 1 - x + \frac{1}{2}x^2}$ (5%)

8. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot x^n}{(n+1) \cdot \ln(n+1)}. \quad (10\%)$$